Department Of Mathematics

Saurashtra University
Rajkot

SYLLABUS
M. Sc. (Mathematics)
(CBCS)
With effect from June – 2017 (Sem – 3 and Sem – 4)

(Reaccredited "A" Grade by NAAC)
(CGPA 3.05)

DEPARTMENT OF MATHEMATICS

Course Structure and Scheme of Examination
For Choice based Credit System (CBCS)

(With effect from June – 2017 (Sem – 3 and Sem – 4))

Semester 3

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester End Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 3001	Prog. In C & Numerical Methods	4	4	30	70	100	3
CMT – 3002	Functional Analysis	4	4	30	70	100	3
CMT – 3003	Number Theory 1	4	4	30	70	100	3
CMT – 3004	Discrete Mathematics	4	4	30	70	100	3
EMT – 3011 OR EMT – 3021	Differential Geometry **OR** Sp. Theory of Relativity and Tensor Analysis	4	4	30	70	100	3
PMT – 3001	Practical (Comp. Applications)	4	8	-	100	100	3
Total		**24**				**600**	

Semester 4

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester end Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 4001	Linear Algebra	4	4	30	70	100	3
CMT – 4002	Integration Theory	4	4	30	70	100	3
CMT – 4003	Number Theory 2	4	4	30	70	100	3
CMT – 4004	Graph Theory	4	4	30	70	100	3
EMT – 4011 OR EMT – 4021 OR EMT – 4031 OR EMT – 4041	Financial Mathematics **OR** General Theory of Relativity & Cosmology **OR** Commutative Ring Theory **OR** Introduction to Mathematical Cryptography	4	4	30	70	100	3
PMT – 4001	Practical (Numerical Methods with Programming)	4	8	-	100	100	3
Total		**24**				**600**	

* CMT – Core Subject, EMT –Elective Subject, PMT - Practical

♦ Passing Standard is 40% in Internal as well as in external examinations for all the courses.

♦ Student will have to clear internal as well as external examinations. (i.e. internal examination with minimum 40% and external examination with minimum 40% is compulsory) and student can earn credits mentioned against each course.

♦ There will be two internal examinations in each course and average of both the examinations will be considered.

M.Sc. SEMESTER 3

Sub. Code: **CMT-3001**
Core Sub. 1: **Prog. In C & Numerical Methods**

Unit 1
Constants, variables, C tokens, keywords, identifiers, declaration of variables, operations and expressions, managing input and output operations and formatted output.

Unit 2
Decision making and branching statements like – if then else, if then switch, go to and loops, jump in loops

Unit 3
One or two dimensional array and their initialization, handling of character strings, User defined functions, structure, unions, pointers and file management in C.

Unit 4
Iterative methods introduction, beginning an iterative method, method of successive bisection, method of false position, Newton-Raphson iterative method, secant method, method of successive approximation, comparison of iterative methods, solution of polynomial equation.

Unit 5
Solution of simultaneous algebraic equations introduction, Gauss elimination method, ill conditioned equations, refinement of the solution obtained by Gaussian elimination, Gauss-Seidel iterative method, comparison of direct and iterative methods. Interpolation introduction, Lagrange interpolation, difference tables.

Reference Books:-

1. Introductory methods of Numerical analysis by S S Sastry, Prentice Hall of India, 1998.
2. Computer Oriented Numerical Methods by V. Rajaraman, Prentice Hall of India, 1994.
3. Programming in C, by E. Balagurusami units 2 to 12.

M.Sc. SEMESTER 3

Sub. Code: **CMT-3002**
Core Sub. 2: **Functional Analysis**

Unit 1
Normed linear spaces, Banach spaces, Quotient space of a normed linear spaces and its completeness, bounded linear transformations, normed linear spaces of bounded linear transformations, dual spaces with examples.

Unit 2
Weak convergence in normed linear spaces, equivalent norms, Riesz lemma, Basic properties of finite dimensional normed linear spaces and compactness, weak convergence in normed linear spaces, reflexive spaces.

Unit 3
Uniform Boundedness theorem and its consequences, open mapping theorem, closed graph theorem, Hahn-Banach theorem for normed linear spaces, compact operations, solvability of linear equations in Banach spaces, the closed range theorem.

Unit 4
Inner product space, Hilbert space, orthonormal sets, Bessel's inequality, complete orthonormal sets, Parseval's identity, structure of Hilbert spaces, projection theorem, Riesz representation theorem for bounded linear functional on Hilbert spaces, reflexivity of Hilbert spaces.

Unit 5
Adjoint of an operator on a Hilbert space, self – adjoint, Normal, Unitary, Positive and Projection operators on Hilbert spaces, abstract variation boundary – value problem, the generalized Lax-Milgrem theorem.

This course is covered by relevant portions from the text "**Introductory Functional Analysis with Applications**", John Wiley and Sons, Newyork, 1978.

Reference Books:-

1. Bachman G. and Warici L, Functional Analysis, Academic Press, 1966.

2. Convway J. B., A Course in Functional Analysis, Springer-verlag, Newyork, 1990.
3. Krishnan V. K. , Text Book of Functional Analysis; A Problem oriented approach, Printice Hall of India, 2001.
4. Limaye B. V., Functional Analysis, New Age International Pvt. Ltd., 2001.
5. Simmons G. F., Introduction to Topology and Modern Analysis, McGraw – Hill book company, Newyork, 1963.
6. Tayor A. E., Introduction to Functional analysis, John Wiley and Sons, Newyork, 1958.

M.Sc. SEMESTER 3

Sub. Code: **CMT-3003**
Core Sub. 3: **Number Theory - 1**

Unit 1
Divisibility, Prime Numbers.

Unit 2
Congruences, Linear Congruences and their solutions, Chinese Remainder Theorem, Degree of a Congruence relation and related theorems.

Unit 3
Primitive rules and related Theorems and Examples, Related Congruences and their solutions.

Unit 4
Largest Integer functions and related results, Arithmetic Functions.

Reference Books:-

1. THE THEORY OF NUMBERS (Authors: Ivan Niven ,Herbert S. Zuckerman, Hugh L. Montgomery)
2. NUMBER THEORY (Authors: Z. I. Borevich and I. R. Shafarevich)
3. AN INTRODUCTION TO THE GEOMETRY OF NUMBERS (Authors: J. W. S. Cassels)
4. HISTORY OF THE THEORY OF NUMBERS (Authors: L. E. Dickson)

M.Sc. SEMESTER 3

Sub. Code: **CMT-3004**
Core Sub. 3: **Discrete Mathematics**

Unit 1

Semigroups and Monoids, Homomorphism of Semigroups and Monoids, Products and Quotients of semigroups, Fundamental theorem of Homomorphism of Semigroups, Subsemigroups and submonoids. Relations, Transitive Closure and Warshall's Algorithm

Unit 2

Lattices as partially ordered sets, Properties of Lattices, Lattices as algebraic systems, Sublattices, Direct product and Homomorphisms of Lattices, Some Special Lattices,Finite Boolean Algebras, Functions on Boolean Algebras, Karnaugh Map Method.

Unit 3

Languages and Grammars, Finite State Machines,Semigroups, Machines and Languages, Moore Machines, Simplification of Machines, Moore Machines and Regular Languages, Kleene's Theorem, Pumping Lemma, Nondeterministic Finite State Automata.

Unit 4

Propositions and Logical operations, Truth tables, Conditional statements and Logical Equivalence, Quantifiers, Rules of Inference.

Unit 5

Elements of Coding Theory, The Hamming Metric, The Parity-Check and Generator Matrices, Group Codes: Decoding with Coset Leaders, Hamming Matrices.

Reference Books:-

1. Grimaldi,R.P, Discrete and Combinatorial Mathematics,3rd Edition, Addison-Wesley Publishing Company, 1994.
2. Johnsonbaugh,R., Discrete Mathematics, Pearson Education,First Indian Reprint,2001.
3. Kolman,B, Busby,R.C., Ross,S.C., Discrete Mathematical Structures, 5th Edition, Pearson Education,2006.

4. Lawson,M.V., Finite Automata, Chapman and Hall/CRC Press, 2004.
5. Tremblay, J.P., Manohar,R., Discrete Mathematical Structures with Applications to Computer Science, Tata-McGraw Hill Publishing Company Limited, New Delhi,21st Reprint, 2004.

M.Sc. SEMESTER 3

Sub. Code: **EMT-3011**

Elective Sub. 1: **Differential Geometry**

Unit 1

Local theory of curves, space curves, examples. Planar curves, Helices, Frenet – Serret apparatus. Existence of space curves, involutes and evolutes of curves.

Unit 2

Local theory of surfaces – parametric patches on surface. First Fundamental form and arc length.

Unit 3

Normal curvature, Geodesic curvature and Gauss formulae, Shape operator L^p of a surface at a point, vector field a curve.

Unit 4

Second and third fundamental forms of a surface, Weingarten map, principal curvatures, Gaussian curvature, mean and normal curvatures.

Unit 5

Riemannian curvatures, Gauss theorem of Egregium, isometry groups and fundamental existence theorem for surfaces.

Reference Books:-

1. R. S. Milman and G. D. Parker, Elements of Differential Geometry, Prentice – Hall, 1977.
2. B. O' Neil, Elements of Differential Geometry, Academic Press, 1966.
3. M. Docermo, Differential Geometry of curves and surfaces, Prentice – Hall, 1976.
4. J. A. Thorpe, Introduction to Differential Geometry, Springer – Verlag.
5. S. Sternberg, Lecture notes on Differential Geometry, Prentice – Hall, 1964.

M.Sc. SEMESTER 3

Sub. Code: **EMT-3021**
Elective Sub. 2: **Special Theory of Relativity and Tensor Analysis**

Unit 1

- Newtonian Relativity (Galilean Transformation)
- Lorentz transformation
- Michelson – Morley experiment

Unit 2

- Length Contraction
- Time dilation
- Relativistic law of addition of velocities
- Equivalence of mass and energy
- Problems related to above topic

Unit 3

- Tensor Algebra
- Vector field in affine and Riemann space

Unit 4

- Christoffel Symbols
- Tensor Analysis

Books:-

1. Related topics of Unit 1 and Unit 2 will be covered from the book "**Special Relativity**" by W. Rindler. Pub.: Oliver and Bosed.
2. Related topics of Unit 3 and Unit 4 will be covered from the book "**Introduction to General Relativity**" by R. Adler, M. Basin, M. Schiffer. Pub.: Mc.Graw Hill Kogakusha Ltd.

Reference Books:-

1. The Special theory of Relativity – Benerji and Benarjee. Pub.: Prentice Hall India Ltd.
2. Essential Relativity – W. Rindler. Pub. Springer Verlag.

M.Sc. SEMESTER 4

Sub. Code: **CMT – 4001**
Core Sub. 1: **Linear Algebra**

Unit 1
The Algebra of linear transformations, Characteristic roots, Matrices.

Unit 2
Canonical Forms: Triangular Form, Nilpotent linear transformations, Invariants of a nilpotent linear transformation.

Unit 3
Canonical Forms: The primary decomposition theorem, Jordan Form, Rational canonical Form.

Unit 4
Trace and Transpose, Determinants, Cramer's rule, Cayley-Hamilton theorem, a quick review of inner product spaces, Hermitian, Unitary and Normal transformations.

Unit 5
Real Quadratic Forms, Sylvester's law of inertia, Bilinear Forms, Symmetric Bilinear Forms, Skew-Symmetric Bilinear Forms, Groups preserving Bilinear Forms.

Reference Books:-

1. N. Herstein, **Topics in Algebra**, 2/e, Wiley Publication, 1975. (For Unit 1 to Unit 4)
2. K. Hoffman & R. Kunze, **Linear Algebra**, 2/e, Prentice Hall of India, New Delhi, 1992. (For Unit 5)

M.Sc. SEMESTER 4

Sub. Code: **CMT-4002**
Core Sub. 2: **Integration Theory**

Unit 1
Measures spaces, Measurable functions, integration, general convergence theorems.

Unit 2
Signed measures, Positive sets, negative sets, null sets and their properties, Hahn-Decomposition Theorem, mutually singular measures, Jordan-Decomposition for a signed measure.

Unit 3
Measure absolutely continuous with respect to another measure, Radon-Nikodym theorem for measure and for signed measure, Lebesgue decomposition theorem, outer measure on a set, algebra of sets, Caratheodary extension theorem.

Unit 4
Product measure, structure of measurable sets in the product measure space, Fubini's theorem, Fonelli's theorem, $L^P(k)$ and Riesz Representation theorem for bounded linear functional on $L^P(k)$, Baire measure on the real line, Lebesegue Stieltjes integral of Borel measurable function with respect to monotonically increasing function.

Unit 5
Locally compact Hausdorff spaces, Baire and Boral measures, continuous functions with compact support, regularity of measures on locally compact Hausdorff spaces, integration of continuous functions with compact support, Riesz Markov-theorem.

Reference Books:-

1. H. L. Royden, Printice Hall of India, Third edition, 1987.
2. G. de Barre, Measure Theory and Integration, Wiley Eastern Limited, 1981.
3. P. R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950.
4. W. Rudin, Real and complex analysis, Tata McGraw Hill Publishing Company, Second Edition, 1974.

5. S. K. Berberian, Measure and Integration, Chelsa Publishing Company, Newyork, 1965.
6. K. Rana, An Introduction to Measure and Integration, Narosa Publishing House, 1997.

M.Sc. SEMESTER 4

Sub. Code: **CMT-4003**
Core Sub. 3: **Number Theory - 2**

Unit 1
Farey Fractions, Irrational numbers, Farey Fractions and Approximation of Irrationals by Rationals.

Unit 2
Continued Fractions(Finite and Infinite), Approximations of Irrationals by Rationals, Hurwitz's Theorem.

Unit 3
Periodic Continued Fractions, Pell's Equations.

Unit 4
Diophantine Equations, Pythagorean Triplets, Some other Examples.

Reference Books:-

1. THE THEORY OF NUMBERS (Authors: Ivan Niven ,Herbert S. Zuckerman, Hugh L. Montgomery)
2. NUMBER THEORY (Authors: Z. I. Borevich and I. R. Shafarevich)
3. AN INTRODUCTION TO THE GEOMETRY OF NUMBERS (Authors: J. W. S. Cassels)
4. HISTORY OF THE THEORY OF NUMBERS(Authors: L. E. Dickson)

M.Sc. SEMESTER 4

Sub. Code: **CMT-4004**
Core Sub. 4: **Graph Theory**

Unit 1
Graph, degree of a vertex, path, circuit, connected and disconnected graphs, components, adjacency and incidence matrix.

Unit 2
Euler circuits, Euler graph, Hamiltonian Paths and circuits.

Unit 3
Trees and their characterizations, Cut-Sets and Cut-Vertices

Unit 4
Planar Graphs, Kuratowski's two graphs, Different representation of planarity, Detection of Planarity.

Unit 5
Coloring of graphs, chromatic number, chromatic polynomial, the four color problem matching

Unit 6
Graph theory in Operation Research: transport networks, extension of Max-Flow, Min-Cut theorem, minimal cost flows.

The syllabus is a covered from chapters 1 & 2 (for quick review), Chapter 3 (3.1 to 3.6), 4 (4.1 to 4.6), 5 (5.1 to 5.5), 8 (8.1 to 8.4) and 14(14.1 to 14.3) from **"Graph theory with application to Engineering & computer science"** by Narsingh Deo, Prentice – Hall of India New Delhi.

Reference Books:-

1. Graph theory by F. Harary, Addision – Wesley, 1969.
2. Graph Theory and Its application by J. Gross and J. Yellen, CRC Press, 2000.
3. Introduction to Graph Theory by D. B. West, 2/e, Prentices – Hall of India, 2001.
4. A textbook of Graph Theory by R. Balakrishnan and K. Ranganathan, Springer, 2012.

M.Sc. SEMESTER 4

Sub. Code: **EMT-4011**
Elective Sub. 1: **Financial Mathematics**

Unit 1

Basic option theory, Types of options, interest rates and present value, Asset price

Unit 2

Random walk, Ito's lemma, Black-Sholes model, arbitrage theorem, option values

Unit 3

The Black – Sholes formulae, hedging the practice, partial differential equations and Black – Sholes formulae.

Unit 4

Variations in Black – Sholes model to include dividends as well as forward and future contracts, American Options.

Reference Books:-

1. P. Willmontt, S. Howison and J. Dewynne, the Mathematics of Financial Derivatives, Cambridge Univ. Press, 1995.
2. Sheldon M. Ross, An elementary introduction to Mathematical Finance, Cambridge Univ. Press, 2003.

M.Sc. SEMESTER 4

Sub. Code: **EMT-4021**

Elective Sub. 2: **General Theory of Relativity & Cosmology**

Unit 1: The Gravitational Field Equation in Empty Space

- Criteria for the field equations.
- The Riemann curvature tensor and its properties.
- The Bianchi identities.

Unit 2: The Schwarzschild solution and its consequences, experimental tests of General Relativity

- The Schwarzschild solution
- The Schwarzschild solution in isotropic co-ordinates
- The General Relativistic Kepler problem and the perihilc shift of Mercury.
- The trajectory of light ray in Schwarzschild field.
- The Schwarzschild radius, Kruskal co-ordinates and the Black hole.

Unit 3: The Kerr Solution

- The Schwarzschild and Kerr solution
- The Kerr solution and Rotation.

Relevant topics will be covered from "**Introduction to General Relativity**". – By R. Adees, M. Bazin & M. Schiffer.

Reference Books:-

1. Essential Relativity – W. Rindler. Pub.: Springer Verlag
2. General Relativity and Cosmology – J. V. Narlikar, Mc-Millan India Ltd.
3. An Introduction to Cosmology – J. V. Narlikar, 3rd edition, Cambridge University Press.

M.Sc. SEMESTER 4

Sub. Code: **EMT – 4031**
Elective Sub. 3: **Commutative Ring Theory**

Unit 1
Rings and ring homomorphisms, Ideals, Quotient rings, Zero-divisors, Nilpotent elements, Units, Prime ideals and Maximal ideals, Nilradical and Jacobson radical, Operations on ideals, Extension and contraction.

Unit 2
Modules and module homomorphisms, Submodules and quotient modules, Operation on Submodules, Direct sum and product, Finitely generated modules, Exact sequences, Rings and modules of fractions, Local properties, Extended and contracted ideals in rings of fractions.

Unit 3
Primary ideals, Primary decomposition, First uniqueness theorem, Second uniqueness theorem, Integral dependence, The Going-Up theorem, Integrally closed integral domains, The Going-Down theorem, Valuation rings.

Unit 4
Noetherian modules, Artinian modules, Composition series of a module, Noetherian rings, Hilbert's basis theorem, Primary decomposition in Noetherian rings.

Unit 5
Artin rings, Structure theorem for Artin rings, Discrete Valuation rings, Dedekind domains, fractional ideals.

Text Book:-

Introduction to Commutative Algebra by M. F. Atiyah and I. G. Macdonald, Addison-Wesley Publishing Company, 1969. (Chapter 1 to Chapter 9)

Reference Books:-

1. N. Bourbaki, **Commutative Algebra**, Springer – Verlag, New York, 1985.

2. O. Zariski & P. Samuel, **Commutative Algebra** Volume I, Van Nostrand, Princeton, 1958.
3. D. G. Northcott, **Lessons on rings, modules and multiplicities**, Cambridge University Press, 1968.
4. D. Eisenbud, **Commutative Algebra with a view toward Algebraic Geometry**, Graduate Texts in Mathematics 150, Springer – Verlag, New York, 1995.

M.Sc. SEMESTER 4

Sub. Code: **EMT – 4041**
Elective Sub. 4: **Introduction to Mathematical Cryptography**

Unit 1
Modular arithmetic, the language of rings and fields, finding multiplicative inverses in $\mathbb{Z}/n$, Fermat's little theorem, the primitive root theorem for $\mathbb{F}p$.

Unit 2
The basic idea of public key cryptography, Diffie – Hellman key exchange and the ElGamal cryptosystem. Language for measuring the complexity of algorithms, and lengths of running times. Attempts to break codes by solving the Discrete Logarithm Problem: brute force attacks, the collision method, and the Pohlig - Hellman algorithm. The Chinese Remainder Theorem.

Unit 3
Euler's formula for powers in $\mathbb{Z}/(pq)$, and the RSA cryptosystem. How to find large primes: the Prime Number Theorem and some Monte Carlo Methods (e.g. the Miller-Rabin test). Algorithms for factoring large integers: Pollards $p-1$ algorithm.

Unit 4
Elliptic curves. Smoothness, the point at infinity, the group law. Using elliptic curves for cryptography. Classification of finite abelian groups. Integer factorization using elliptic curves (Lenstra's method).

Text Book:-

An Introduction to Mathematical Cryptography by Jeffrey Hoffstein, Jill Pipher & Joseph H. Silverman, Springer – Verlag, 2008. (Chapters 1, 2, 3 & 5)

Reference Books:-

1. Paul Garrett, **Making, Breaking Codes: Introduction to Cryptology**, 1/e, Prentice Hall, (2000).
2. Douglas Stinson, **Cryptography: Theory and Practice**, 2/e, Chapman & Hall/CRC, (2002).

3. J. H. Silverman, **A friendly introduction to number theory**, Prentice Hall, (2001).
4. J. Menezes, P. C. Van Oorschot & S. A. Vanstone, **The handbook of Applied Cryptography**, CRC Press, (1996).
5. Neal Noblitz, **Algebric Aspects of Cryptography**, Springer, (1998).
6. J. A. Buchmann, **Introduction to Cryptography**, Springer – Verlog, (2000).