# **Department of Mathematics**

# Saurashtra University Rajkot

# SYLLABUS M. Sc. (Mathematics)

With effect from June-2016



(Reaccredited "A" Grade by NAAC) (CGPA 3.05)

# **DEPARTMENT OF MATHEMATICS**

# <u>Course Structure and Scheme of Examination</u> <u>For Choice based Credit System (CBCS)</u>

(With effect from June-2016)

> Course: M.Sc. (Mathematics)

➤ Eligibility for the admission:- **B.Sc.** (**Mathematics**)

> Duration:-Two years

#### Semester 1

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester End Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 1001	Algebra 1	4	4	30	70	100	2.5hrs
CMT – 1002	Real Analysis	4	4	30	70	100	2.5hrs
CMT – 1003	Topology 1	4	4	30	70	100	2.5hrs
CMT – 1004	Theory of Ordinary Differential Equations	4	4	30	70	100	2.5hrs
CMT – 1005	Seminar and Problem Session	4	4	-	100	100	-
EMT – 1001	Classical Mechanics 1	4	4	30	70	100	2.5hrs
Total		24				600	

#### Semester 2

Subject Code	Title of the Course	Course Credits	No. of Hrs. Per Week	Weightage For Internal Examination	Weightage For Semester End Examination	Total Marks	Duration Of Semester end Exam in hrs.
CMT – 2001	Algebra 2	4	4	30	70	100	2.5hrs
CMT – 2002	Complex Analysis	4	4	30	70	100	2.5hrs
CMT – 2003	Topology 2	4	4	30	70	100	2.5hrs
CMT – 2004	Methods in Partial Differential Equations	4	4	30	70	100	2.5hrs
CMT – 2005	Seminar and Problem Session	4	4	-	100	100	-
EMT – 2001	Classical Mechanics 2	4	4	30	70	100	2.5hrs
Total		24	_			600	-

# M.Sc.(Mathematics) - SEMESTER 1

CMT - 1001 Algebra- I

CMT - 1002 Real Analysis

CMT - 1003 Topology- I

CMT - 1004 Theory of Ordinary Differential Equations

CMT - 1005 Seminar and Problem Session

EMT - 1001 Classical Mechanics- I

# M.Sc.(Mathematics) - SEMESTER 2

CMT - 2001 Algebra- II

CMT - 2002 Complex Analysis

CMT - 2003 Topology- II

CMT - 2004 Methods in Partial Differential Equations

CMT - 2005 Seminar and Problem Session

EMT - 2001 Classical Mechanics- II

- \* CMT Core Subject, EMT Elective Subject, PMT Practical
  - ◆ Passing Standard is 40% in Internal as well as in external examinations for all the courses.
  - ◆ Student will have to clear internal as well as external examinations. (i.e. internal examination with minimum 40% and external examination with minimum 40% is compulsory) and student can earn credits mentioned against each course.
  - ◆ There will be two internal examinations in each course and average of both the examinations will be considered.

Sub. Code: **CMT-1001** 

Core Sub. 1: Algebra- 1

#### Unit 1

Basic concepts of group theory:

Group, abelian group, cyclic group, normal subgroup, quotient group, permutation group, Group isomorphism and their properties, Cayley's theorem, Automorphisms of groups.

#### Unit 2

Direct Products, Finitely Generated Abelian Groups, Invariants of a finite Abelian Groups, Sylow Theorems.

#### Unit 3

Quick look at basic ring theory:

Euclidean ring, Quotient ring and zero divisors, Ideals, principal ideal, maximal ideal and prime ideal, Homomorphisms of ideals, Sum and Direct Sum of Ideals, Nilpotent and Nil Ideals.

#### Unit 4

Euclidean domains, Principal Ideal Domains, Unique Factorization Domains and Polynomial Rings over UFD. Polynomial rings over rational field, irreducible polynomials, Einstein irreducibility criterion.

- 1) P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, **Basic Abstract Algebra**, Second Edition, Cambridge University Press, 1995.
- 2) M. Artin, Algebra, Prentice-Hall of India Private Ltd., New Delhi, 1994.
- 3) J. A. Gallian, Contemporary Abstract Algebra, Fourth Edition, Narosa Publishing House, New Delhi, 1999.
- 4) N. S. Gopalakrishnan, University Algebra, New Age International Private Ltd. Publishers, New Delhi, Sixth Reprint, 1998.
- 5) I. N. Herstein, Topics in Algebra, Second Edition, Wiley Pub., New York, 1975.

Sub. Code: **CMT-1002** Core Sub. 2: **Real Analysis** 

#### Unit 1

Algebra of sets,  $\sigma$ -algebra of sets, Borel sets, Lebesgue outer measure, Measurable sets and Lebesgue measure, A nonmeasurable set, Measurable Functions, and Littlewood's three principles.

#### Unit 2

Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure, The integral of a nonnegative function, The general Lebesgue integral, and Convergence in measure.

#### Unit 3

Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, and Absolute continuity.

#### Unit 4

 $lL^p$  spaces, The Holder's inequality, The Minkowski's inequality, and Convergence and completeness.

The course is covered by Chapter 1 Section 4, Chapter 2 Section 7, Chapter 3 (full), Chapter 4(full), Chapter 5 (Sections 1 to 4), and Chapter 6 (Sections 1 to 3) from the book Real Analysis by H. L. Royden, Third Edition, PHI Learning Private Limited (2009) New Delhi.

- 1. Real Analysis by N. L. Carothers, Cambridge University Press (2000).
- 2. Measure Theory and Integration by G de Barra, Wiley Eastern Limited, First Wiley Eastern Reprint (1987).
- 3. Real Analysis by V. Karunakaran, Pearson (2012).
- 4. Fundamentals of Real Analysis by S. K. Berberian, Universitext, Springer (1999).
- 5. An introduction to Measure and Integration by I. K. Rana, Narosa Publishing House, New Delhi.

Sub. Code: **CMT-1003** Core Sub. 3: **Topology -1** 

#### Unit-1

Topology, Open sets and closed sets, Finer and Coarser topology, Basis for a topology, Simply ordered topology.

#### Unit-2

Subspace topology, Product topology, Continuous functions, Homeomorphism.

#### Unit-3

Limit points, Closure, Interior points and interior, Convergent Sequence.

#### Unit-4

Metric topology, Uniform convergence, Topology of R<sup>n</sup>.

#### Unit-5

Connectedness, Local connectedness, Components, Path connectedness

The Course is covered by following Chapter 1, 2 and 3(Upto article 25) of Topology-A first course, J. M. Munkres, Printice Hall of India (2000).

- 1) Introduction to Topology and Modern Analysis G. F. Simmons, Tata McGraw Hill edition-2004.
- 2) General Topology by S. Willard, Addison Wesley Publishing Company (1970).

Sub. Code: CMT-1004

Core Sub. 4: Theory of Ordinary Differential Equations

#### **Unit 1:** Linear System of Differential Equations

The existence and uniqueness theorem, Linear Homogenous systems, Linear Non-Homogenous systems, Nonlinear system of first order equations.

#### **Unit 2:** Linear System with constant coefficients

The exponential of matrix, Eigen values and eigen vectors of matrices, calculation of fundamental matrix, two dimentional linear systems, some population problems, an electric circuit.

# **Unit 3:** Series solutions of Linear Differential Equations

Review of properties of power series, second order linear equations with analytic coefficients, theorem on solutions in power series, singular points of linear differential equations, solutions about a regular singular point, exceptional cases, the Bessel equation and some properties of Bessel functions, singularities at infinity, irregular singular points with an introduction to asymptotic expansions

# **Unit 4:** Existence theory

Existence of solutions, uniqueness of solutions, continuation of solutions, the non linear simple pendulum, existence theory for system of first order equations and higher order equations, linear systems, dependence on initial conditions.

# **Unit 5:** <u>Laplace Transforms</u>

Linearity, existence theorem, Laplace transform of derivatives and integrals, shifting theorem, differentiation and integration of transforms, convolution theorem, inverse Laplace transform, solution of Ordinary Differential equations and integral equations.

This course is covered by "Ordinary Differential Equations", First course by R. Brauer and J. A. Nohel, Second edition, Benjamin Inc.

- 1) Ordinary Differential Equations by G. Birkoff and G. C. Rota, Second edition, Ginn and Co(1995)
- 2) Introduction to Ordinary Differential Equations by E. A. Coddington, Prentice Hall of India, 1996.
- 3) Elements of Ordinary Differential Equations by M Golom and M. E. Shinks, Second Edition, McGraw-Hill Books Co., 1965.
- 4) Theory and Problems of Differential Equations by F. Ayers, McGraw Hill, 1972.
- 5) Advanced Engineering Mathematics by E. Kreyzig, John Willey and Sons, 2002.

Sub. Code: **EMT-1001** 

Elective Sub.1: Classical Mechanics -1

# **Unit 1:** D'Alemberts principle and Lagrange's Equations

- Conservation theorem for linear momentum and angular momentum for a particle.
- Conservation theorem for linear momentum and angular momentum for a system of particles.
- Classification of dynamical system.
- Constraints.
- Virtual displacement and principle of virtual work.
- Generalized force in holonomic system
- Mathematical expression for principle of virtual work
- D'Almbert's principle
- Lagrange's equation for holonomic system
- Lagrange's equation for conservative non-holonomic system
- Problems on above topics

# **Unit 2**: Variational principle and Lagrange's equations

- Variational principle
- Calculus of variations
- Hamilton's principle
- Derivation of Hamilton's principle from Lagrange's equation
- Derivation of Lagrange's equations from Hamilton's principle

- Cyclic co-ordinates
- Conservation theorems
- Problems on above topics

#### **Unit 3**: Two Body Central force problem

- Reduction to equivalent one body problem
- The equations of motion and first integrals
- The equivalent one dimensional problem and classification of orbits
- The inverse square law of force.

# **Unit 4:** Equations of Motion and Rigid bodies

Independent co-ordinates of rigid bodies, generalized coordinates of a rigid bodies, Euler angles, Cayley-Klein parameters and related quantities, components of angular velocity along the body set of axes, Euler's theorem on the motion of a rigid body, rate of change of a vector, the coriolis force, Euler's equations of motion for a rigid body, finite rotations, infinitesimal rotations.

The course is covered by the above topics from the book:

- **1. Classical Mechanics** by H. Goldstein, 2<sup>nd</sup> Edition, Narosa Publishing House
- **2. Classical Mechanics** by C. R. Mondal, Prentice Hall of India Pvt. Ltd.

Sub. Code: **CMT-2001** 

Core Sub. 1: Algebra- 2

#### Unit 1

Division ring and Field, Extension fields, algebraic and transcendental extensions, Splitting fields, Normal extensions, Multiple roots, Finite fields, Separable extensions.

#### Unit 2

Automorphism fixed fields, Galois extension, Fundamental theorem of Galois Theory, Fundamental theorem of Algebra.

#### Unit 3

Modules (Definitions and examples), Submodules and Operation on modules

#### Unit 4

Homomorphisms of modules and quotient modules, completely reducible modules, finitely generated modules.

- 1) P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Second Edition, Cambridge University Press, 1995.
- 2) M. Artin, Algebra, Prentice-Hall of India Private Ltd., New Delhi, 1994.
- 3) J. A. Gallian, Contemporary Abstract Algebra, Fourth Edition, Narosa Publishing House, New Delhi, 1999.
- 4) N. S. Gopalakrishnan, University Algebra, New Age International Private Ltd. Publishers, New Delhi, Sixth Reprint, 1998.
- 5) I. N. Herstein, Topics in Algebra, Second Edition, Wiley Pub., New York, 1975.

Sub. Code: **CMT-2002** 

Core Sub. 2: Complex Analysis

#### Unit 1

The extended complex plane and its spherical representation, analytic functions, bilinear transformations, their properties and classifications, Branches of many valued functions with special reference to arg z, log z and z<sup>a</sup>, elementary Riemann surfaces, definition and properties of conformal mapping.

#### Unit 2

Riemann – Steiltjes integral and its properties, line integral and its properties, fundamental theorem of calculus for line integral, Leibnitz rule, Taylor's theorem, Cauchy's integral formula dn Cauchy's theorem for analytic functions on an open disc, winding number of a closed rectifiable curve with respect to a point outside the curve and its properties, Cauchy's integral formula first version and second version, Cauchy's theorem first version, second version, third version and forth version.

#### Unit 3

Cauchy – Goursat theorem, Moreras theorem, Cauchy's inequality, entire functions, Llouville's theorem, identity theorem, fundamental theorem of algebra, maximum modulus theorem and minimum modulus theorem.

#### Unit 4

Schwartz lemma, meromorphic functions, argument principle, Rouche's theorem, Open Mapping Theorem, Inverse function theorem.

#### Unit 5

Isolated singularities, classifications of singularities, Laurent's series, residue theorem, evaluation of integrals. This course is covered by relevant portions from the text "Functions of One Complex Variable" by John B. Conway, Third Edition, Springer International Student Edition, Narosa Publishing House.

- 1) Complex Analysis by L. V. Ahlfors, International Student Edition, Mc Graw Hill Book Company, 1979.
- 2) Complex Analysis by Karunakaran, Second Edition, Narosa Publishing House, 2006.
- 3) A First Course in Complex Analysis with Applications by Dennis G. Zill and Patrik D. Shanahan, Second Edition, Jones & Bartlett Student Edition, 2010.
- 4) Complex Analysis by S. Lang, Addison-Wesley, 1977.
- 5) Foundations of Complex Analysis by S. Ponnusamy, Narosa Publishing House, 1977.
- 6) Fundamentals of Complex Analysis with Applications to Engineering and Science by E. B. Saff and A. D. Snider, Third Edition, Pearson Education.
- 7) Notes on Complex Function Theory by D. Sarasan, Hindustan Book Agency, 1994.

Sub. Code: CMT-2003

Core Sub. 3: Topology- 2

<u>Unit – 1:</u>

Separation Axioms:  $T_1$  – Spaces,  $T_2$  – Spaces (Hausdorff Spaces).

**Unit – 2:** 

Separation Axioms: Regular Spaces, Completely Regular Spaces, Normal

Spaces.

**Unit – 3:** 

Compact Spaces, Locally Compact Spaces, Limit Point Compact Spaces.

<u>Unit – 4:</u>

Sequentially Compact Spaces, Compact Metric Spaces.

<u>Unit – 5:</u>

Complete Metric Spaces.

- 1) Topology A First Course, J.R.Munkres, Prentice Hall of India (2000). Chapter 3 (Article no. 26 to 29), Chapter 4 (Article no. 31,32,33 and 35) and Chapter 7 (Article no. 43)
- 2) General Topology by S.Willard, Addison Wesley Publishing Company (1970)
- 3) Introduction to Topolgy & Modern Analysis, G.F.Simons, Tata Mcgraw Hill (2004)

Sub. Code: **CMT-2004** 

# Core Sub. 4: Methods in Partial Differential Equations

#### Unit 1

Surfaces and Curves in three dimensions, Simultaneous differential equations of the first order and the first degree in three variables, Methods of solutions of dx/P = dy/Q = dz/R, Orthogonal trajectories of a system of curves on a surface. Pfaffian Differential forms and equations, Solution of Pfaffian differential equations in three variables, and Miscellaneous problems.

#### Unit 2

Partial differential equations, Origins of First-order partial differential equations, Linear equations of the first order, Integral Surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces.

#### Unit 3

Non-linear partial differential equations of the first order, Charpit's method, Special types of first order equations, Solutions satisfying the given conditions, Jacobi's method, and Miscellaneous problems.

#### Unit 4

The origin of second order equations, Linear partial differential equations with constant coefficients, and Equations with variable coefficients.

This course is covered by the relevant portions from the book 'Elements of Partial Differential Equations' by Ian Sneddon, McGraw-Hill Book Company.

- 1. Partial Differential Equations by F. John, Narosa Publishing Company, New Delhi, 1979.
- 2. Elementary Course in Partial Differential Equations by Amarnath, Narosa Publishing House, New Delhi, 1997.

Sub. Code: **EMT-2001** 

Elective Sub. 1: Classical Mechanics -2

#### **Unit 1:** The Rigid Body Equations of Motion

Angular momentum and kinetic energy of motion about a point, the inertia tensor and moment of inertia, the heavy symmetrical top with one point fixed.

# **Unit 2:** Special Relativity in Classical Mechanics

The basic program of special relativity, The Lorentz transformation, Lorentz transformations in real four dimensional spaces, Further descriptions of the Lorentz transformation, Covariant four – dimensional formulations, The force and energy equations in relativistic mechanics.

# **Unit 3:** <u>Hamilton's equation of Motion</u>

Derivation of Hamilton's equation of motion, Routh's procedure, derivation of Hamilton's equation from Hamilton's Principle, principle of least action, problem related to above topics.

# **Unit 4:** Canonical transformations and Generating functions

Poisson's brackets and their properties, Hamilton-Jacobi theory, problem related to above topics.

The course is covered by the above topics from the book:

- **1. Classical Mechanics** by H. Goldstein, 2<sup>nd</sup> Edition, Narosa Publishing House
- **2. Classical Mechnaics** by C. R. Mondal, Prentice Hall of India Pvt. Ltd.